A Reactive Agent Based Approach to Facility Location: Application to Transport

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ABSTRACT

Facility location problem concerns the positioning of facilities such as train stations, bus-stops, fire stations and schools, so as to optimize one or several objectives. The review of different facets of this problem shows a real interest for transportation systems. Since the location model decisions are an influencing factor for the relevance and the attractiveness of transportation services. This paper contributes to research on location problem by proposing a reactive multiagent model to deal with a classical variation: the p-median problem, where the objective is to minimize the weighted distance between the demand points and the facilities. The proposed approach has a physical inspiration. It is based on potential fields technique, especially using attractive and repulsive forces between agents and their environment. The optimization of the system is then obtained from a global self-organization of the agents (the facilities). The efficiency of the proposed approach is confirmed by computational results based on a set of comparisons with the K-means clustering technique. Particularly, the approach is evaluated on the problem of bus-stops positioning in a real bus-network.

Categories and Subject Descriptors

I.2.11 [**Distributed Artificial Intelligence**]: Multiagent systems; G.1.6 [**Optimization**]: Global optimization

General Terms

Theory, Experimentation

Keywords

Facility location, P-median Problem, Multiagent systems, Reactive agents, Potential Fields, Optimization, Transport

1. INTRODUCTION

The facility location problems have witnessed an explosive growth in the last four decades. As Krarup and Pruzan [15] point out, this is not at all surprising since location policy is one of the most profitable areas of applied systems analysis where ample theoretical and applied challenges are offered. The term facility is used in its broadest sense. It refers to entities such as bus-stops, train stations, schools, hospitals, fire stations, etc. A wide range of facility location models exists [26]: set covering, p-center, p-median, etc. The general problem is, then, the location[1] of new facilities to optimize some objectives such as distance, travel time or cost and demand satisfaction.

The operations research community devoted a strong interest to location problem analysis and modeling. This is due to the importance of location decisions which are often made at all levels of human organization. Then, such decisions are frequently strategic since they have consequential economic effects.

However, location problems are often extremely difficult to solve, at least optimally [9] (classified as NP-Hard). Furthermore, there does not exist a generic solution that is appropriate for all potential or existing applications.
There exists some works based on genetic algorithms, branch and bound, greedy heuristics, etc. These approaches are not easily adapted for dynamic systems where the system constraints or data change. This is a real limitation since most of real problems are subject to change and dynamics. To deal with this lack of flexibility and robustness, we adopt a multiagent approach which is known to be well suited for dynamical problems [8].

This paper proposes a multiagent approach for the facility location problem based on reactive agents. To our knowledge, no reactive agent-based approaches have been already used to deal with this problem. The choice of a multiagent approach provides several advantages. First, multiagent systems are well suited to model distributed problems. In such systems, several entities evolving/moving in a common environment have to cooperate to perform collective and local goals. Second, even if the multiagent approach does not guarantee to find optimal solution, it is able to find satisfying ones without too much computational cost [27]. Through this paper we show that the reactive multiagent

[1] Deployment, positioning and siting are used as synonyms

approach can be an interesting new way for optimization in positioning problems. Then, it provides satisfying solutions in addition to other assets as flexibility, modularity and adaptability to open systems. In our approach, agent behavior is based on the combination of attractive and repulsive forces. The key idea is that agents are attracted to the demands and repulsed by other agents.

This paper is structured as follows: section 2 presents the facility location problems. Then, section 3 details the proposed multiagent approach. Section 4 presents experimental evaluation on two cases: (1) positioning stations on the France map, and (2) positioning bus-stops for the real case of the city of Belfort (France). Section 5 is devoted to the model discussion. Then, the last section gives some conclusions and perspectives.

2. THE FACILITY LOCATION PROBLEM

In the parlance of literature, the general facility location problem consists in locating new facilities to optimize some objectives such as distance, travel time or cost, demand satisfaction. In the following section, we present several variants of facility location problems that can be tackled by our approach.

2.1 Variants of the problem: an overview

There are four components that characterize location problems [21]: (1) a space in which demands and facilities are located, (2) a metric that indicates distance (or other measure as time) between demands and facilities, (3) demands, which must be assigned to facilities, and (4) facilities that have to be located. There exists two types of location problems: continuous and discrete ones.
The problem is continuous when the facilities to be sited can generally be placed anywhere on the plane or on the network. In discrete location problems the facilities can be placed only at a limited number of eligible points.
A non-exhaustive list of facilities problems includes: p-center, p-median, set covering, maximal covering.

- Set Covering Location Problem (SCLP) [26]: the objective is to locate the minimum number of facilities required to cover all the demand nodes.

- Maximal Covering Location Problem (MCLP) [4]: the objective of the MLCP is to locate a predetermined number of facilities such that the covered demand is maximized with the assumption that there may not be enough facilities to cover all the demand.

- p-median problem [10, 11, 3]: this problem locates p facilities that will serve n demand points in some space. The space can be an euclidean plane or a road network. The objective is to minimize the weighted distance between the demand points and the facilities. In this paper we will particularly focus on the p-median problem.

- p-center [10, 11]: the p-center problem addresses the problem of minimizing the maximum distance between a demand and its closet facility, given that we are siting a predetermined number of facilities.

There exists several other classes of location problems [6]: dynamic location problems, where the time dimension is introduced, these problems recognize that the parameters (e.g. demand) may vary over time; stochastic location problems where the problem parameters are not known with certainty; multiobjective location problems that consider multiple, often conflicting, objectives, etc.

2.2 Solving approaches

We have seen in the previous section that there is a wide range of facility location problem variants. Their mathematical formulations are well known [6]. However, formulating is only one step of analyzing a location problem. The other step and the most challenging one is to find optimal solutions.

Typically, the possible approaches to such a problem and especially to the p-median problem, consist in exact methods which allow to find optimal solutions. A well-known example of methods is branch and bound [24]. However, these solutions are quickly inefficient for very complex problems, i.e. with hundreds of constraints and variables. Then obtaining optimal solutions for these problems requires colossal computational resources. This justify the NP-hardness of the p-median problem [9].
Another category of methods are proposed for the p-median problem. These methods, known as heuristics, allow to find good solutions, but do not guarantee finding the optimal one(s): Greedy heuristics [5]; Genetic algorithms [2, 13]; Improvement heuristics [19, 25]; Lagrangean relaxation [7], etc.
However, these approaches have several drawbacks such as the computational cost (huge population size and long convergence time, for example in genetic algorithms); their rigidity and their lack of robustness and flexibility. Particularly, for dynamic problems characterized by the change of the problem constraints, optimization criteria, etc.

This paper explores another possible heuristic which is based on multiagent systems. For the following of the paper we will focus on this approach. After presenting the p-median problem and the reactive multiagent systems, we detail our approach.

3. A REACTIVE APPROACH FOR THE CONTINUOUS P-MEDIAN PROBLEM

In this section we present a reactive agents based solution to the p-median problem. Our model relies on the Artificial Potential Fields (APF) technique. This technique is generally used for decentralized coordination of situated agents [1, 18].

3.1 Problem statement

We consider the problem of continuous stations positioning to illustrate our approach. It consists to locate a fixed number of stations (train stations or bus-stops) such that the whole environment space may be used for locating stations. The objective is to minimize the distance between users demand and the stations.
The problem is expressed as follow [20]:
A = the set of demand points in the plane $\Re^2$ (or more generally $\Re^n$) indexed by a
W_a = a positive weight assigned to each demand
P = the maximum number of facility lo locate
The problem is to find a subset X of P within a feasible

region $S \subset \Re^2$, such that:

$$\min_{X \subset S} F_A(X) \quad (1)$$

$$F_A(X) = \sum_{a \in A} W_a \ . \ \min_{x \in X} d(x, a)$$

Subject to:

$$\sum_{x \in X} x \leq p \quad (2)$$

The objective function (1) minimizes the demand-weighted distance. Constraint (2) stipulates that at most p facilities are to be located.

3.2 Reactive agent model

An agent can be viewed as an entity that is able to perceive its environment and to act according to its own decisions [8]. The decision making process can be complex, as in cognitive architectures [12], or more simple as in reactive ones.

3.2.1 Potential Fields Based Approach

Reactive agents have simple behaviors based on reaction to stimuli coming from the environment. Intended to handle basic behaviors, their architectures are based on simple routines without abstract reasoning [27]. Such a scheme is more appropriate to deal with numerous agents having collective processes. Agents have numerous interactions between them and their environment in a stimulus-response way to collectively organize the whole system [8].
Reactive agents have been deployed in several fields, such as collective robotics [1], complex systems simulation, distributed problem solving [23], web agents construction [8].
In many works, the behavior of reactive agents is based on the Artificial Potential Field technique. This method has several inspirations (physical, biological, etc.). The concept was introduced in Lewin's topological psychology [16]. The key idea is that the human behavior is controlled by a force field generated by objects or situations with positive or negative values or valences.

During the past decade, potential field theory has gained popularity among researchers in the field of autonomous robots [14] and especially in robot motion planning thanks to their capability to act in continuous domains in real-time. By assigning repulsive force fields to obstacles and an attractive force field to the desired destination [22], a robot can follow a collision-free path via the computation of a motion vector from the superposed force fields [14, 28]. However, the APF technique is limited by a well known drawback: local minima [1]. Indeed, adding attractive and repulsive fields can produce areas where forces are equilibrated. Then, an agent that uses potential fields to move can be trapped in such places. The originality of our approach relies on the fact that we do not try to avoid such local minima. At the opposite, we exploit them as interesting places where facilities are located at the balance of different influences.

3.2.2 Agent characteristics and behaviors

As facilities are elements to be placed in the environment, we consider them as reactive agents. The environment is defined by a finite and continuous space. Demands, which are static data of the problem, are defined as an environment characteristic.

Typically, in a transportation network the objective is to increase accessibility for the customers by satisfying their transportation demands. A customer is covered if the next station is within a specified distance, called the covering radius. This objective is traduced in our model by an attraction behavior, i.e. station agents are attracted by demands. This attraction behavior must be balanced with repulsive forces between agents to avoid agents gathering. The solution we adopt ensures the repartition of agents in the environment. Indeed, stations should not be too far nor too close to each other (it is the inter-agent distance constraint). Agent behavior is then based on the reaction of two types of influences: attractive and repulsive. These influences are generated through agents perceptions following distances separating agents and obstacles, demands, etc.
Each agent has a local perception of the environment and of other agents. As attraction and repulsion influences are considered separately, we define two perception radius: attractive and repulsive radius (see Fig.1).

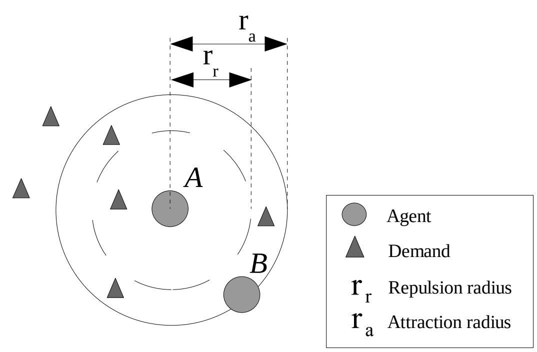

Figure 1: Perception radius of an agent

In order to satisfy the local coverage, we set attraction forces as induced by demands. The attraction is an interaction between agents and their environment. Each agent perceives the demand within its attraction radius (see Fig.2). Considering one demand point, an attractive force is defined from the agent towards the demand. It is expressed as a vector which intensity is proportional to the demand weight and to the distance between the agent and the demand.
Formally, for an agent A perceiving a demand D with weight W_D:

$$\vec{F}_{D/A} = W_D \ . \ \overrightarrow{AD} \quad (3)$$

The influence of the attraction decreases when the agent moves towards the demand. Thus, if the agent attains the demand the attraction behavior is inhibited. Furthermore, if an agent is subject to two attractive forces (from two different demands), it will be more attracted towards the biggest demand. Then, it will move towards a balance point. This point is defined as the place where the two attraction forces are equilibrated.

The global attraction force is the sum of all forces (between the agent and each perceived demand). Formally, the global attraction force undergone by an agent A is computed as follows:

$$\vec{F}_{demands/A} = \frac{\sum_{i=1}^{n} \vec{F}_{i/A}}{n} \quad (4)$$

n is the number of demands perceived by the agent A through its attraction radius ($n = 5$ in Fig.2). The demand is indexed by i.

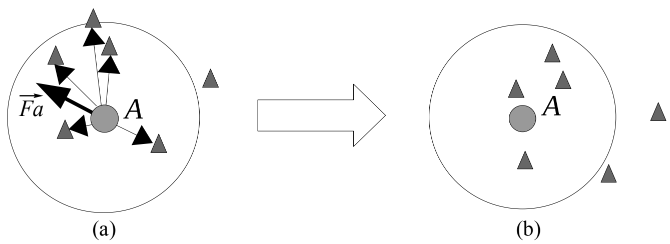

Figure 2: (a) Attraction to demands (b) The agent moves to the balance point

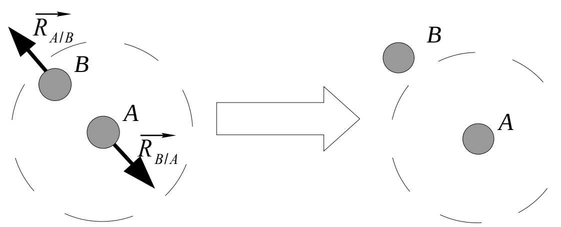

Figure 3: Repulsion between two agents A and B

If we consider only attractions, agents should move towards common locations. Such a process is sub-optimal and does not respect constraints on distances separating facilities in location problems. Then, to ensure an efficient agents repartition, we define repulsive forces between agents. Each agent generates a repulsion force which is inversely proportional to the inter-agent distance (see Fig.3). Consequently, agents will be subject to this force until the inter-agent distance is ensured. Formally the repulsive force induced by an agent B on an agent A is expressed as follow:

$$\vec{R}_{B/A} = \frac{\overrightarrow{BA}}{\left\|\overrightarrow{AB}\right\|^2} \tag{5}$$

Then, the global repulsive force undergone by an agent A is computed as follows:

$$\vec{R}_{agents/A} = \frac{\sum_{j=1}^{m} \vec{R}_{j/A}}{m} \tag{6}$$

m is the number of agents perceived by the agent A. These agents are indexed by j.
Contrary to the attraction influence, the repulsion is an interaction between agents.
The agent behavior is defined as the weighted sum of both global attraction and repulsion forces. Formally, for an agent A, it is expressed as follows:

$$\overrightarrow{Move} = \alpha\ \vec{F}_{demands/A} + (1-\alpha)\ \vec{R}_{agents/A} \tag{7}$$

The coefficient α allows to favour either the attraction or the repulsion. The global solving process is presented in Algorithm 1. The initialization (step 1) and the fitness computation (step 9) will be explained in the next section.

Algorithm 1 The system behavior

```
 1: Initialization of Agent positions
 2: while Fitness in progress do
 3:   for all Agents do
 4:     Attraction computation
 5:     Repulsion computation
 6:     Move computation
 7:     Move execution
 8:   end for
 9:   Fitness computation
10: end while
```

4. EXPERIMENTATIONS

After exposing the principle of our approach, we evaluate the model on two case studies. The first one consists in positioning stations (for instance train stations) on a continuous map without environment constraints. The second one consists in locating bus-stops on an existing bus-network (i.e. a constrained environment).

4.1 Application to stations location

In the first case we consider a continuous environment corresponding to the France map presented in Fig.4 (400x400 size). It contains the demand weights which are values between 0 and 255 (randomly generated). These weights are represented as a gradation from black color (255) to white color (0). The initialization step is performed with a random positioning. Parameters values are: $\alpha = 0.5$, $r_a = 25$, $r_r = 20$.

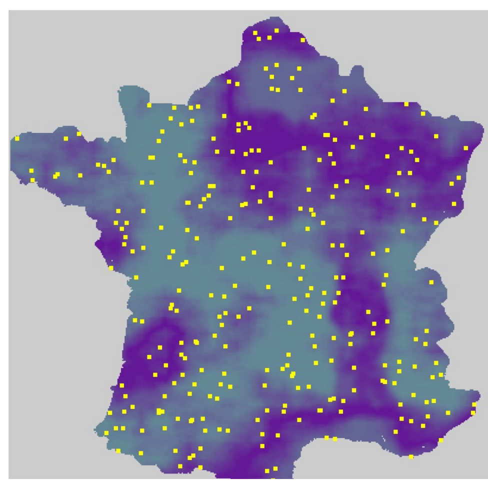

Figure 4: Demand representation (dark areas) and random initialization of station locations

When the algorithm starts, facility agents (the points in Fig.4) move towards demands while avoiding other agents. The repartition is ensured thanks to the combination of attractive and repulsive influences. The system iterates until it attains a global equilibrium state (converge to a stable state). In practice, the system converges to a finite number of close states (see Fig.6). In Fig.5 we notice the final state to which the system converges. We observe the stations repartition which is characterized by an intensification of the agents in areas where demands is high. This result

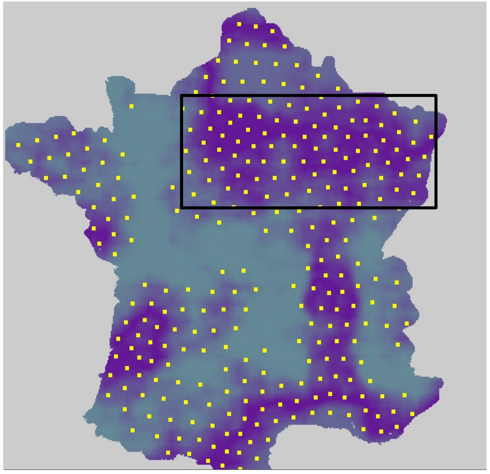

Figure 5: Final result (since the iteration 41)

is clearly visible inside the rectangular area (see Fig.5). It is also visible that all facilities respect the minimal distance between them.

We have compared the performance of our multiagent model with the K-means clustering technique. K-means algorithm is a well known technique that computes very good solutions [17]. It allows to classify or to group objects based on attributes/features into K number of groups. The grouping is done by minimizing the sum of distances between data and the corresponding cluster centroid (see Algorithm 2).

Algorithm 2 The K-means clustering

1: **repeat**
2: Place K points into the space represented by the objects that are being clustered.
3: Assign each object to the group that has the closest centroid. When all objects have been assigned, recalculate the positions of the K centroids as weighted barycenters.
4: **until** The centroids no longer move.

Comparisons are made according to a global fitness index expressed by the formula (8) and corresponding to the mean distance between each demand and the nearest station:

$$Fitness = \frac{\sum_{ij} V_{ij} \ . \ d(C_{ij}, x_{ij})}{\sum_{ij} V_{ij}} \tag{8}$$

Where:
V_{ij} = the demand at point x_{ij}
$d(C_{ij}, x_{ij})$ = the distance between the point x_{ij} and the nearest station C_{ij}

Comparisons are carried out on different number of stations, as shown in Table 1. For each stations number, about 40 tests have been executed.

The fitness values obtained by applying the multiagent approach are very close to the k-means ones. The deviation between the two approaches is small and it is inversely proportional to the number of stations.

Fig.6 plots the evolution of the fitness values for 400 stations. We can see that the fitness decreases until the convergence to a constant value. Here, the convergence is attained rapidly: since the 41 th iteration.

All the experimentations have shown that the agents systematically converge to a stable location. It corresponds to a global balance between attraction to demands and inter-agents repulsive forces. These results show that the reactive approach is an interesting heuristic technique to deal with such optimization problems. We now present its application to a real transport problem.

Table 1: Comparison with k-means clustering

Fitness: minimal values					
Stations	**50**	**80**	**100**	**150**	**200**
MultiAgent	16,592	12,501	11,187	9,164	7,945
K-means	15,556	12,253	10,965	9,010	7,820
Deviation	6.65%	2.02%	2.02%	1.7%	1.5%

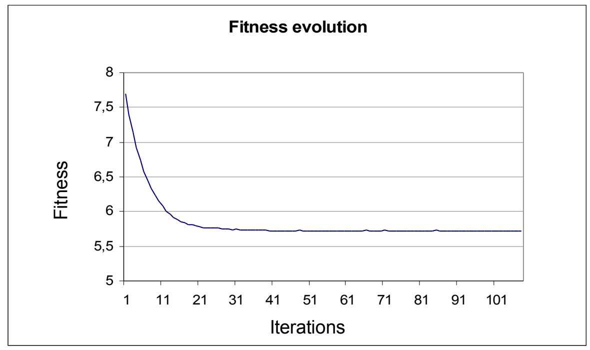

Figure 6: The fitness evolution for the case study with 400 station agents

4.2 Application to bus-stops location

In this section we apply our model to the bus-stops positioning on the bus-network of the city of Belfort (east of France, 60,000 inhabitants). We dispose of the real bus-network, see lines structure in Fig.7, and the real values of demands which correspond to inhabitants density per quarter. In Fig.7, dark areas characterize important demands. This example introduces an important constraint for facilities. While moving, bus-stop agents must remain on the lines. The integration of this new constraint does not need change in the model. Its adaptation concerns only the projection of the move vector on the bus-lines (i.e. the move vector is transformed so as the agents move along lines).

The initial bus-stop positions are computed randomly (see Fig.7). Agents can be anywhere on the lines network. Each line has a predetermined number of bus-stops. White points in Fig.7 correspond to bus-stop agents. Terminus agents are fixed and are not subject to attractive and repulsive influences. The organization of bus-stop agents is ensured thanks to the previous influences.

Fig.8 shows the final state to which the system converges. We observe the bus-stops repartition which is characterized by an intensification of the agent number in areas where demand is high. The fitness index has been computed for the case of Belfort by using the formula 8. Fig.9 plots the fitness values which decrease until the convergence to a static state. Convergence is attained rapidly: since the 24 th iteration.

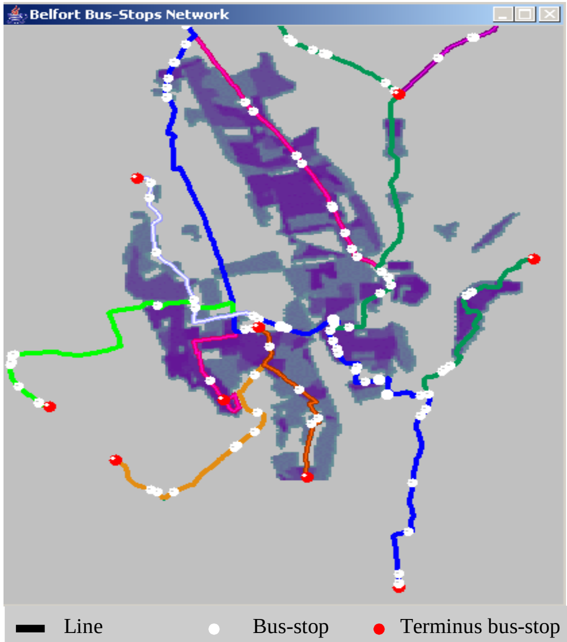

Figure 7: Belfort bus-network; random initialization of the bus-stop agents

The optimal number of this fitness is 175 meters. In other words, for a person demand, the nearest bus-stop is situated at an average distance of 175 meters.

5. DISCUSSION

The previous experimentations allow to point up some observations on the proposed model. The obtained solutions are globally satisfying considering the fitness values. This last are quickly obtained, i.e. few iterations are necessary to reach a stable state.
It is worth noting that the agent initialization has an influence (even if it is slight) on the solution quality.

For each specific application, the parameters setting can be an important step. The results quality can depend on parameter values. However, the model is based only on three parameters: attraction and repulsion radius, and the weight combination of influences (α in formula 7). Attraction and repulsion radius depend on the considered application. Generally, the attraction radius is defined by the coverage distance and the repulsion one is defined by the maximal distance between two facilities. Concerning the parameter α, it allows to express a preference for the satisfaction of the demand constraint or the inter-agent distance constraint.

The existing solutions for facility location are not easily adaptable when the problem changes, particularly, for dynamic systems characterized by a variation of the problem constraints or data. The proposed multiagent approach allows to tackle this lack of flexibility. We have shown that specific constraints can be taken into account without changing the agent behaviors. For instance, when considering the bus-lines network we have just forced the agents to remain on the lines (cf. section 4.2). Other dynamic changes may concern the environment structure (e.g. demands, bus-lines network), the facilities number, etc.

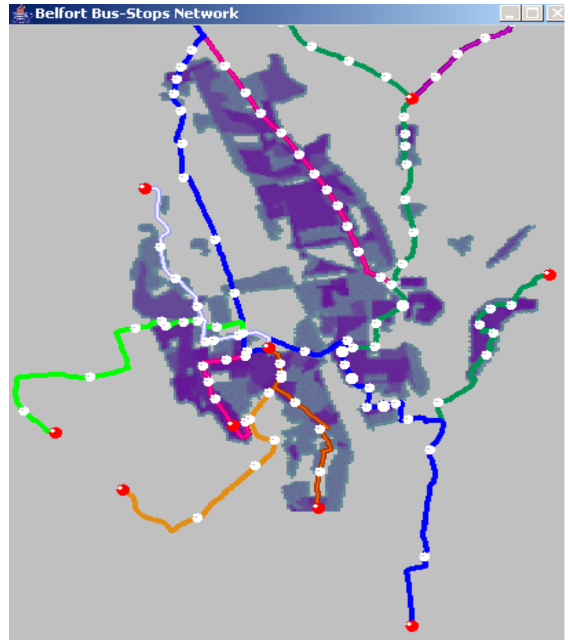

Figure 8: Execution final result

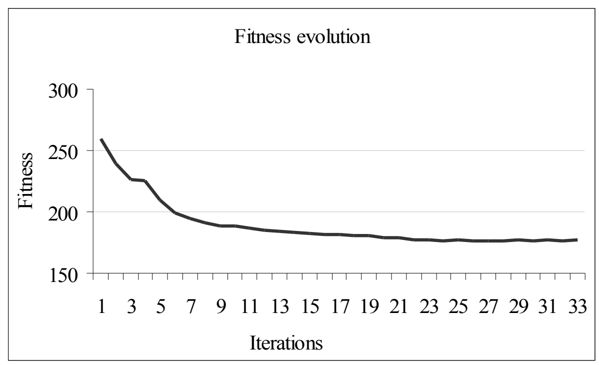

Figure 9: The fitness evolution

The proposed model is not limited to a specific facility location variant. It can be adapted through changes of the agent behaviors.

Concerning the design of bus-lines network, we extend our model to deal with more complex transport constraints. Particularly, we are currently working on tools computing connections in the network. The approach consists to agentify the lines and to consider the connections as the result of line interactions. These interactions lead to the merging of close bus-stop agents into connections.

6. CONCLUSIONS

This paper has presented a reactive multiagent approach for the facility location problem. Facilities, which are modeled as agents, move in artificial potential fields induced by the demand and close agents. The agent behavior is a combination of reactions to attractive and repulsive influences. Local minima, which must be avoided in the artificial potential fields approach are exploited in our model as balance points between demands.

The relevance of the approach was proved by its application to transport: the location of stations and bus-stops. Then, it was compared with the k-mean clustering technique. The evaluation criteria concerns the deviation from the K-mean clustering and the convergence time. They show that the reactive multiagent approach is a very interesting perspective for such optimization problems.

Future works deal, first, with a more complete evaluation of the global system convergence. Then, we seek to apply our approach to another problematic in location problems: the dimensioning problem. It consists to optimize the number of facilities to locate, since each new facility increases the location cost. We obtain a multicriteria problem. We then propose to add two behaviors based on the creation and the removal of agents to tackle the facility location and number optimization problem.

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